TTA-45026 Financial Engineering

Exam

November 14, 2016

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This is a closed-book exam, a calculator allowed. You can answer in English or in Finnish. Good luck!

Question 1.

a) Explain Incomplete Markets (FRA) (1 p)

b) Explain Control Variates in Monte Carlo simulation (3 p)

c) How do you replicate a chooser option with vanilla European call and put options? (2 p)

Question 2.

a) Suppose that Black-Scholes assumptions hold and that you represent a *bank* that has a *short* position on a vanilla European *call* option. Explain how to implement delta hedging strategy in order to mitigate the risk associated to the underlying stock diffusion. Just demonstrate the sequence of necessary actions (step-by-step). (4 p)

b) Suppose that there will be three possible states in our one-step economy (i.e. trinomial tree): stock's current price is 100 EUR and after one month the stock price will be worth 120 EUR, 100 EUR, or 80 EUR. The interest rates are zero, the strike price is 100 EUR, and time to maturity is one month (i.e. one time-step). Assume unlimited shorting, fractional quantities, and no transaction costs nor taxes. To hedge an option, you can use a bond and a stock only.

   Is the price of a European call option unique and arbitrage-free? If yes, please calculate it and explain carefully how you got it. If not, what can be said about the option price and about the markets? (2 p)

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Question 3. Consider a *Down-and-Out Asian Arithmetic Average Strike Call* option that ceases to exist if the asset price, $\{S_t; 0 \leq t \leq T\}$ reaches a barrier $H < S_0$ and otherwise pays

$$\max(S_T - \bar{S}, 0),$$

where

$$\bar{S} = \frac{1}{n} \sum_{i=1}^{n} S_{t_i}$$

is an arithmetic average over $1, 2, \ldots, n$ equally spaced observations and $t_n = T$ is the maturity time. Additionally, S_0 is the current stock price and $S_T = S_{t_n}$ the terminal stock price. Suppose that the risk-free interest rate, r, and volatility, σ, are constants, and that the stock price follows geometric Brownian motion.

Please give a pseudo code that prices the above contract using Monte Carlo methods with *antithetic* variates. (6 p)