TTA-45026 Financial Engineering

Exam
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This is a closed-book exam, a calculator allowed. You can answer in English. Good luck!

Question 1.

a) Explain *Martingale* (1 p)

b) Explain *Control Variates* in Monte Carlo simulation (3 p)

c) How do you replicate a *chooser option* with vanilla European call and put options? (2 p)

Question 2.

Please answer **two** of the following four questions:

a) Assume that the stock prices follow

$$S_t = \mu S_t dt + \sigma S_t dW_t. \tag{1}$$

Apply Ito's lemma to derive process $f_t = 1/S_t$. [Remember that $D(1/g(x)) = -g'(x)/(g(x)^2)$]. (3 p)

b) Assume the stock prices follows Eq (1). What is the probability that the terminal price S_T is more than or equal to the target price X, $\mathbb{P}[S_T \geq X|S_0]$, where S_0 is the initial stock price? Show that $\mu < \frac{1}{2}\sigma^2$, then an increase in the length of the time period T decreases this probability. (3 p)

c) You observe that the *continuously* compounded yield of a two-year discount bond (zero-coupon bond) is 8% and the corresponding one-year yield is 7.5%. What is the implied forward rate for the second year, i.e. using our notation, the value of $F(0;1,2)$? Suppose that the time measure $\tau_{ij} = T_j - T_i$. (3 p)

Question 3. Consider a European-style exotic derivative with a payoff

$$\max\left(\sqrt{S_{1,T} \times S_{2,T}} - K, 0\right),$$

where T is the maturity time, K the strike price, and $S_{1,t}$ and $S_{2,t}$ are the prices of two stocks at time t, which follow geometric Brownian motions with correlated random increments with a constant correlation coefficient ρ_{12}. Also all other Black-Scholes assumptions apply. Please give a pseudo code that prices the above contract using the antithetics variates. (6 p)

r=0,05

dt = 1/252

for i = 1 to n_sim:
S_one = start
S_two = start
r = 0.
for j = 1 to T/dt:
eps = randn(2) || ε1 and e12
S_one = S_one * exp(r - 0.5 sigma