TTA-45046 Financial Engineering

Exam
September 13, 2018

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This is a closed-book exam, a calculator allowed. Please answer in English.
Good luck!
Mathematical Appendix is enclosed.

Question 1.

a) Explain Swaptions (2 p)

b) Explain Shout Option (2 p)

c) Let η be annual return that satisfies $S_T = S_0 e^{\eta T}$, $T > 0$. Express the mean and standard deviation of η. What happens to the standard deviation of η if the length of the investment period T increases? (2 p)

Question 2.

a) Assume that the stock prices follow

$$S_t = \mu S_t dt + \sigma S_t dW_t. \tag{1}$$

Apply Ito's lemma to derive process $f_t = \ln(S_t)$. [Remember that $D(\ln(x)) = 1/x$]. (3 p)

b) Assume the stock prices follows Eq (1). Express the probability that the terminal price S_T is more than or equal to the target price X, $\mathbb{P}[S_T \geq X|S_0]$, where S_0 is the initial stock price. Show that $\mu < \frac{1}{2}\sigma^2$, then an increase in the length of the time period T decreases this probability. (3 p)

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Question 3. Consider a *Up-and-out Digital Put* option that ceases to exist if the asset price, $\{S_t; 0 \le t \le T\}$ reaches a barrier $H > S_0$ and otherwise pays \$1 if $S_T \le K$ at the maturity T. Here S_0 is the current stock price, S_T the terminal stock price, and K the strike price. Suppose that the risk-free interest rate, r, and volatility, σ, are constants, and that the stock price follows geometric Brownian motion.

Please give a pseudo code that prices the above contract using Monte Carlo methods with *antithetic* variates. (6 p)