TTA-45036 Introduction to Financial Engineering and Derivatives Markets

Exam
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This is a closed-book exam, a non-programmable calculator allowed. Answer in English. Good luck!

Question 1. Explain the following concepts and terms:

a) Incomplete markets (1 p)

b) Implied volatility (1 p)

c) Forward contract (1 p)

d) Draw a payoff curve (diagram) of the portfolio of following options (3 p):

- A long call option with strike 10 EUR
- Two short put options with strike 12 EUR

Draw payoff curves for call, for two puts, and for the portfolio. The underlying price can vary between 5 and 17 EUR.

Question 2.

a) What is wrong with the following expressions for price a call option in arbitrage free markets (two things)

$$C_t = \max[E_t(S_T) - K, 0]\frac{1}{(1+k)^T},$$

where S_T is the price of the underlying stock at maturity T, K is the strike price and k is the required risk-adjusted rate of return of the underlying stock. $E_t[.]$ is conditional expectation. (3 p)

b) Suppose that Black-Scholes assumptions hold and that you represent a bank that has a short position on a vanilla European call option. Explain how to implement delta hedging strategy in order to mitigate the risk associated to the underlying stock diffusion. Just demonstrate the sequence of necessary actions (step-by-step). (3 p)

Question 3.

a) Show that no-arbitrage bounds for the European put option prices are

$$P(t,T) < KD(t,T)$$
$$P(t,T) > (KD(t,T) - S(t))^{+},$$

where $S(t)$ is stock price at time t and $P(t,T)$ is the price of a put at time t with maturity time T, $T > t$. Moreover, $D(t,T)$ is a discount factor (with risk-free interest rate) from time T to time t. Assume that you can borrow or lend any amount of money at the risk-free interest rate. (2 p)

b) Let the current stock price $S_0 = 1$, strike price $K = 0.96$, continuously compounded interest rates $r = 0.05$, time to maturity $T = 2$ years, and volatility $\sigma = 0.25$. The stock pays no dividends. What is Black-Scholes price of the European call option? Here $\ln(1) = 0$ and $\ln(0.96) = -0.0408$. (1 p)

c) A stock is worth \$10 today and *monthly* return coefficients are $U = 1.2$ and $D = 1/U$ (i.e. stock price will be either $\$10 \times 1.2$ or $\$10/1.2$ after the first month). The continuously compounded risk free interest rate (annual) is 2%. The strike price is \$9.5 and time to maturity 2 months. What is the price of a European call option with two-step binomial tree (with $\Delta t = 1/12$)? Second, suppose that market price for the European call option is \$1. How could one exploit an arbitrage opportunity? (3 p)