

**TTA-45036 Introduction to Financial Engineering and
Derivatives Markets**

Exam

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This is a closed-book exam, a non-programmable calculator allowed. You can answer in English or in Finnish.

If you have submitted a project work earlier in 2017 or 2018, please indicate this clearly in the first page!

Good luck!

Question 1. Explain the following concepts and terms:

- a) Efficient markets (1 p)
- b) Incomplete markets (1 p)
- c) Put option (1 p)
- d) Implied volatility (1 p)
- e) Short position (1 p)
- f) Forward contract (1 p)

Question 2.

- a) Mathematical finance assumes that financial markets do not allow for profitable arbitrage and that the liquid markets price instruments correctly. Why to use mathematical arbitrage-free models to price options and other derivative securities at all if market prices are already assumed to be correct? (3 p)
- b) What is the difference between Black-Scholes implied volatility and historical volatility estimated from time-series? Are you better off using implied volatility or historical volatility to forecast future volatility? Why? (3 p)

Question 3.

- a) Show that no-arbitrage bounds for the European put option prices are

$$P(t, T) < KD(t, T)$$
$$P(t, T) > (KD(t, T) - S(t))^+,$$

where $S(t)$ is stock price at time t and $P(t, T)$ is the price of a put at time t with maturity time T , $T > t$. Moreover, $D(t, T)$ is a discount factor (with risk-free interest rate) from time T to time t . Assume that you can borrow or lend any amount of money at the risk-free interest rate. (2 p)

- b) Let the current stock price $S_0 = 1$, strike price $K = 0.96$, continuously compounded interest rates $r = 0.05$, time to maturity $T = 2$ years, and volatility $\sigma = 0.25$. The stock pays no dividends. What is Black-Scholes price of the European call option? (1 p)
- c) A stock is worth \$10 today and *monthly* return coefficients are $U = 1.3$ and $D = 1/U$ (i.e. stock price will be either $\$10 \times U$ or $\$10 \times D$ after the first month). The continuously compounded risk free interest rate (annual) is 3%. The strike price is \$9.5 and time to maturity 2 months. What is the price of a European call option with two-step binomial tree (with $\Delta t = 1/12$)? Second, suppose that market price for the European call option is \$1. How could one exploit an arbitrage opportunity? (3 p)