TTA-45036 Introduction to Financial Engineering and Derivatives Markets

Exam

February 28, 2017

Juho Kanniainen, Milla Siikanen

This is a closed-book exam, a non-programmable calculator allowed. You can answer in English or in Finnish. Good luck!

Question 1. Explain the following concepts and terms:

a) Efficient markets (1 p)

b) Incomplete markets (1 p)

c) Put option (1 p)

d) Implied volatility (1 p)

e) Short position (1 p)

f) Forward contract (1 p)

Question 2.

a) Mathematical finance assumes that financial markets do not allow for profitable arbitrage and that the liquid markets price instruments correctly. Why to use mathematical arbitrage-free models to price options and other derivative securities at all if market prices are already assumed to be correct? (3 p)

b) What is the difference between Black-Scholes implied volatility and historical volatility estimated from time-series? Are you better off using implied volatility or historical volatility to forecast future volatility? Why? (3 p)

Question 3.

a) Show that no-arbitrage bounds for the European put option prices are

$$P(t,T) < KD(t,T)$$
$$P(t,T) > (KD(t,T) - S(t))^{+},$$

where $S(t)$ is stock price at time t and $P(t,T)$ is the price of a put at time t with maturity time T, $T > t$. Moreover, $D(t,T)$ is a discount factor (with risk-free interest rate) from time T to time t. Assume that you can borrow or lend any amount of money at the risk-free interest rate. (2 p)

b) Let the current stock price $S_0 = 1$, strike price $K = 0.96$, continuously compounded interest rates $r = 0.05$, time to maturity $T = 2$ years, and volatility $\sigma = 0.25$. The stock pays no dividends. What is Black-Scholes price of the European call option? (1 p)

c) A stock is worth \$10 today and *monthly* return coefficients are $U = 1.2$ and $D = 1/U$ (i.e. stock price will be either \$10 $\times$ 1.2 or \$10/1.2 after the first month). The continuously compounded risk free interest rate (annual) is 2%. The strike price is \$9.5 and time to maturity 2 months. What is the price of a European call option with two-step binomial tree (with $\Delta t = 1/12$)? Second, suppose that market price for the European call option is \$1. How could one exploit an arbitrage opportunity? (3 p)